Alexander Mikhailovich Lyapunov (1857–1918)

Alexander Lyapunov created the modern theory of equilibrium stability. Before him, stability problems were typically solved in the first approximation, i.e., by discarding all the non-linear terms of the equations without ascertaining the validity of such a linearization. The foundational work in this area is Lyapunov's doctoral dissertation *The General Problem of Stability of Motion*.

In 1900, as he was preparing for a series of lectures on the theory of probability, Lyapunov suggested a new method: the method of characteristic functions. Having generalized the research by Chebyshev and Markov (Sr.), he proved the central limit theorem in the probability theory under much weaker assumptions than his predecessors. A large series of research papers by Lyapunov concerned the theory of figures of equilibrium of a uniformly rotating fluid whose particles are mutually attracted according to the law of universal gravitation. Before his research, it had been proved that the equilibrium figures for homogeneous fluids are ellipsoidal,[1] and Lyapunov was the first to rigorously prove the existence of close to ellipsoidal equilibrium figures for homogeneous and weakly inhomogeneous fluids.

The great mathematician's grandfather, Vasily Alexandrovich Lyapunov (1778–1847), was a syndic of the Imperial Kazan University Board, i.e., an official representative of the university authorized to conduct its affairs. Vasily Alexandrovich and his wife Anastasia Evseevna had nine children: three sons and six daughters. All the children received a good education.

Mikhail Vasilievich Lyapunov (1820–1868), Alexander's father, graduated from the Mathematics Department of Kazan University with a silver medal in 1839. His main fields of study were mathematics and astronomy. In 1840, he was appointed as the observer-astronomer at the university observatory.

[1] The ellipsoids of Maclaurin (1742) and Jacobi (1831).

After the Kazan fire of 1842, Mikhail Vasilyevich was sent to Pulkovo to oversee the repair of equipment from the university observatory that was damaged in the fire. There, he studied astronomy under Vasily Struve (Friedrich Georg Wilhelm von Struve), Otto Struve (Otto Wilhelm von Struve), Yegor Sabler (Georg Thomas Sabler), and others. In 1845, Lyapunov returned to Kazan to his former position as an astronomical observer, and in 1850, he was appointed director of the Kazan observatory.

In 1853, Mikhail Lyapunov married Sofia Alexandrovna Shipilova, who was then aged 28.

Akexander Lyapunov with his wife, Bolobonovo, 1904. From archives of A.N. Lyapunov.

In 1855, Mikhail Lyapunov was elected a corresponding member of the university, and soon retired. The following year, he was appointed principal of the Yaroslavl Demidov Lyceum, and he and his wife moved to Yaroslavl. They

had a son there, Alexander, born on June 6th (May 25th Old Style[2]) 1857. Alexander spent his early childhood with his mother and brothers in the village of Bolobonovo, Kurmyshsky county, Simbirsk province. His mother's letters, sent during this period, show what a troublesome time it was: serfdom had just been abolished, and it was very difficult for her to build a new relationship with the peasants. She did not know how to deal with this unprecedented situation, and no one around her had any advice for her or experience around such issues.

Mikhail Vasilyevich resigned from his position as headmaster of the Demidov Lyceum in 1864 due to health reasons; the family settled in Bolobonovo in a wooden (one-story, five-room) house, which Mikhail Vasilyevich had just built on the estate received by Sophia Alexandrovna from her parents.

Here, Mikhail Lyapunov had a wonderful "library, replete with works in Russian, German, and French, and ranging across not only subjects like mathematics, astronomy, and the natural sciences, but also philosophy, history, ethnography, political economy, and literature."

Initially, the children's education was supervised by their mother, but from the age of seven, they were taught by their father, who devoted himself entirely to this task after his retirement.

When his father died in 1868, Alexander was 11 and a half years old. He continued his studies in the family of Raphail Mikhailovich Sechenov, an artist, married to his father's sister Ekaterina Vasilievna. Here, he and his cousin, his future wife Natalia Rafailovna, a year younger than him, were trained in gymnasium[3] subjects and learned new languages under the guidance of hired teachers and his aunt Glafira Vasilievna Lyapunova.

Two years later, Alexander, his mother, and his brothers moved to Nizhny Novgorod where he was admitted to the third class of Nizhny Novgorod gymnasium. In the autumn of 1876, having graduated with a gold medal, Alexander entered the physics and mathematics department of St. Petersburg University. At first, he studied at the department of natural sciences, being particularly zealously engaged in chemistry with Dmitri Mendeleev. Just a month later, however, he transferred to the department of mathematics, where Pafnuty Lvovich Chebyshev taught at that time, who, in Alexander's own words, "had a significant influence on the character of [my] subsequent scientific activity with his lectures and advice."

In 1880, Alexander Lyapunov received a gold medal for an essay written on a particular topic in analytical mechanics that was suggested by the faculty, and published two papers on hydrostatics in the *Journal of the Physical and Chemical Society* after making a presentation at a meeting of the Society.

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[2] 'Old Style' refers to the Julian calendar, used by the Russian Orthodox Church. For example, Orthodox Christmas Day occurs on January 7th, whereas the use of the Gregorian calendar means Christmas Day is celebrated on December 25th.

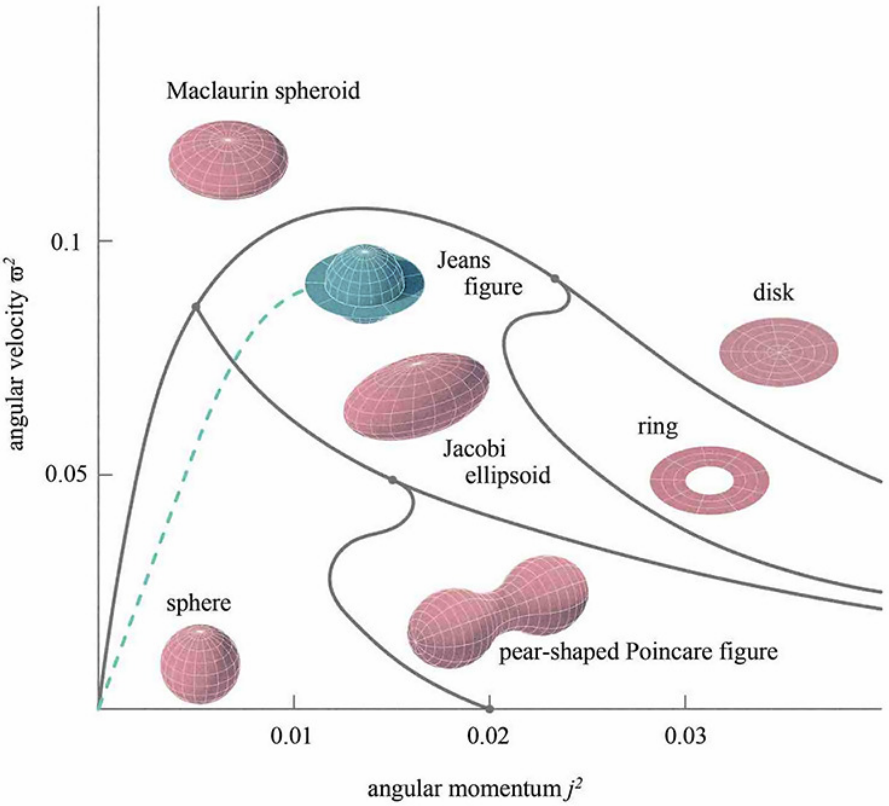
[3] *Gymnasium* is a grammar school.

After his mother died suddenly in 1879, Alexander took over the care of his younger brother Boris. From 1881 to 1885, he and his brother shared a room in the flat of the sister of Ivan Mikhailovich Sechenov,[4] Professor of Physiology. During that time, Alexander liked to work at night. Once a week, the family, including Ivan Sechenov, liked to relax in the company of young students, who gathered on Sundays at Sechenov's sister's flat. In those days, Alexander gave Sechenov lessons in branches of mathematics that he considered especially important for a physiologist. Sechenov delighted in and wholeheartedly supported all of Alexander's scientific successes. For two years, Lyapunov had been diligently working on a problem proposed by Pafnuty Chebyshev. Although he failed to solve Chebyshev's problem, he obtained a closely related result on the stability of the ellipsoids of Maclaurin and Jacobi. In 1885, Alexander Lyapunov defended this work as his thesis for

[4] Ivan Mikhaylovich Sechenov (1829–1905), a Russian physiologist, was one of the originators of objective psychology and the author of the classic Reflexes of the Brain, in which he introduced electrophysiology and neurophysiology into laboratories and the teaching of medicine.

The house in Bolobonovo where Lyapunovs lived after the retirement of Mikhail Vasilyevich. 1904. On the left: Boris Lyapunov, future academician, and philologist. From archives of A.N. Lyapunov.

a master's degree and, in the autumn of the same year, moved to a position as a privatdozent at Kharkov University.



Types of stability figures (ellipsoids of Maclaurin and Jacobi, etc.) in Chebyshev's problem, [6].

Being in Kharkov, Lyapunov taught extensively and strenuously and later considered this period a complete loss to science, although he did obtain a number of remarkable results during this time. His famous version of the central limit theorem in probability theory is also linked to his teaching activities during this period. For the winter holidays, Alexander came to St. Petersburg, where on the 17th of January 1886, he married his cousin Natalia Sechenova, Ivan Sechenov's niece, to whom he was deeply attached from his early childhood.

According to his closest pupil, Vladimir Steklov, Lyapunov worked every day until 4 or 5 AM. He allowed himself almost no entertainment, and when he occasionally appeared (once or twice a year) at a theatre or concert, it was only on the most exceptional of occasions, such as the rare concerts given

by his brother, the renowned composer Sergei Lyapunov. Steklov wrote: "He sometimes gave off the impression of being a silent, frowning, and reserved person because he was so absorbed in his scientific speculations that he looked — but didn't see, listened — but didn't hear, as his father-in-law Raphael Sechenov sometimes kindly laughed at Lyapunov's absent-minded behaviors amongst close friends."

The results obtained by Lyapunov on stability formed the subject of his doctoral dissertation, *The General Problem of the Stability of Motion*, defended at Moscow University in 1892. In 1900, Alexander Lyapunov was elected a corresponding member of the Imperial Academy of Sciences, and in 1901 he was elected an ordinary member of the Applied Mathematics Division. A year later, Lyapunov moved to St. Petersburg. He now had the opportunity to devote himself wholly to science and to work on Chebyshev's problem.

In 1908, Alexander Lyapunov was sent to the Fourth International Mathematical Congress in Rome. He planned to meet Henri Poincaré, with whom he had common scientific interests and a scientific correspondence that began during the last decade of the XIXth century. Unfortunately, due to the poor organization of the congress, these plans were not fulfilled, and he never had the chance to meet Poincaré.

However, Lyapunov did meet some other colleagues in Rome: the French mathematicians Émile Picard, Jacques Hadamard, and Édouard Goursat, and the Italian mathematicians Vito Volterra, Giuseppe Veronese, and Tullio Levi-Civita, to name a few.

Lyapunov published four grand memoirs containing a complete solution to the Chebyshev problem. After Lyapunov's tragic death, he left behind a completed extensive manuscript in which he developed his results.

The work accomplished by Lyapunov during the last 15 years of his life is nothing short of a remarkable feat.

In the summer of 1917, Alexander left for Odessa with the hope that its southern climate would benefit his wife's health, which was severely damaged by tuberculosis. They never returned to Petrograd, where they had left their apartment with all their possessions.

The last year of his life was a tragic one. Alexander Mikhailovich struggled to deliver a series of lectures at Novorossiysk University due to complete exhaustion, his impending blindness (from a cataract), and the increasingly deteriorating condition of his wife.

The news he and his brother Boris received from relatives and colleagues on rare occasions was not cheerful. One day they received word that the beloved house built by their father, where they had spent their childhood, had been burned down, along with their library, by peasants in the area.

On the 31st of October 1918, the tragic ending came. His wife Natalia died; Alexander shot himself and was taken to a surgical clinic with a gunshot wound, where he died on the 3rd of November 1918 without regaining consciousness.

In the note he left behind, he wrote that he wished to be buried in the same grave as his wife.

Lyapunov's work was widely recognized. During his lifetime, he was elected an honorary member of the Universities of St. Petersburg, Kharkov, and Kazan, a foreign member of the Accademia dei Lincei in Rome, a corresponding member of the Academy of Sciences of Paris, a foreign member of the Circolo Matematico di Palermo, an honorary member of the Kharkov Mathematical Society, a full member of the Moscow Mathematical Society, etc. Lyapunov Stability Theory is now studied in universities around the world. In 1969, the USSR Academy of Sciences established the Lyapunov Gold Medal, and after the collapse of the USSR in 1995, the Russian Academy of Sciences established the Lyapunov Prize. In Moscow and Kharkov, there are Lyapunov streets that are named after him.

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